Exploring and Discovering Mathematics

Joel Reyes Noche
jrnoche@mbox.adnu.edu.ph
http://joelnoche.multiply.com/

Department of Mathematics
College of Arts and Sciences, Ateneo de Naga University

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Camarines Sur, Philippines
Exploring mathematics

Who wrote *The Royal Book of Oz*?
Order in Pollock’s chaos

Discovering mathematics

Folding paper in half repeatedly
Triangle inequalities and triangle centers

An open problem

Do all triangles have periodic billiard paths?

The slides for *Exploring mathematics* are based on the talk *Mathematics in the Arts and Sciences* given on June 17, 2008 at the Ateneo de Naga University (AdNU), which was based on the talk *The Role of Mathematics in the Total Development of the Human Person* given on February 9, 2008 at AdNU.

The slides for *An open problem* are from the talk *Periodic Billiard Paths in Triangles* given on February 4, 2012 during the 2012 Bicol Mathematics Conference at AdNU.
Who wrote *The Royal Book of Oz*?

[Binongo, 2003]

L. Frank Baum wrote *The Wonderful Wizard of Oz* in 1900. He died in 1919 while writing the 14th Oz book *Glinda of Oz*. The 15th Oz book *The Royal Book of Oz*, was supposedly written by Baum and ‘enlarged and edited’ by Ruth Thompson. By 1939, Thompson had published the 33rd Oz book. In 2003, Binongo used multivariate analysis and stylometry to resolve the dispute of who really wrote *The Royal Book of Oz*. 
Preparing the data set
Finding 50 variables to describe each text

Step 1: Prepare machine-readable versions of the books.

Baum’s Canon
14 books
623K words

Thompson’s Canon
14 books
568K words

The Royal Book of Oz
15th Oz Book
42K words

Step 2: Assemble the Oz corpus.

Oz Corpus
29 books
1,235K words

Step 3: Select the 50 most frequent function words.

Word$_1$

Word$_2$

...

Word$_{50}$
Choosing the words to study

<table>
<thead>
<tr>
<th>Fifty stylometric variables (word, average rate of occurrence in %)</th>
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<tr>
<td>the</td>
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<td>and</td>
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<td>to</td>
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<td>not</td>
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<td>as</td>
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</tbody>
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Function words (and not content words) were chosen. Auxiliary verbs and personal pronouns were not included. Misspellings were corrected and contractions were expanded.
Reducing 50 dimensions into 2

Each book was partitioned into blocks of 5,000 words. This yielded a matrix of 223 text blocks and 50 words. To help us visualize this, the multivariate statistical technique of principal component analysis was used.

The best two-dimensional approximation is found by rotating the original 50 axes to new axes so that the latter represent directions of decreasing variability.

The 50 principal component (PC) scores are approximated by the first two PC scores. (The 1st PC accounts for 20% of the total variation; the 2nd PC 7%).
Baum’s and Thompson’s canonical works (Oz-related)
Baum’s and Thompson’s’s canonical works (Oz-related)
Component loadings
Baum’s non-canonical works (Oz-related)

- Santa Claus
- Queen Zixi
- John Dough
- Sea Fairies
- Sky Island

Exploring and Discovering Mathematics

- Exploring mathematics
- Who wrote The Royal Book of Oz?
Baum’s non-canonical works (not Oz-related)

- Master Key
- Enchanted Island
Baum’s and Thompson’s short stories

- Baum’s *Magical Monarch of Mo*
- Baum’s *American Fairy Tales*
- Baum’s *Animal Fairy Tales*
- Thompson’s *Wizard of Way-up*
Glinda of Oz (1920), The Royal Book of Oz (1921)
Visitors from Oz by Gardner (1998)

A: 5,000-word text blocks.
Visitors from Oz by Gardner (1998) (continued)

B: 10,000-word text blocks.
No. 5, 1948 by Jackson Pollock
[Vogel, 2006]

In 2006, this painting was reportedly sold for about $140 million, the highest sum ever known at that time to have been paid for a painting.
Fractal analysis of Pollock’s drip paintings


Scientific objectivity proves to be an essential tool for determining the fundamental content of the abstract paintings produced by Jackson Pollock in the late 1940s. Pollock dripped paint from a can onto vast canvases rolled out across the floor of his barn. Although this unorthodox technique has been recognized as a crucial advancement in the evolution of modern art, the precise quality and significance of the patterns created are controversial. Here we describe an analysis of Pollock’s patterns which shows, first, that they are fractal, reflecting the fingerprint of nature, and, second, that the fractal dimensions increased during Pollock’s career.
Fractals are self-similar

[Taylor, 2002]
The fractal character of Pollock’s paintings
[Taylor, 2002]

The painting is scanned into a computer. It is separated into its different colored patterns, then covered with a computer-generated mesh of identical squares. The computer analyzes which squares are occupied and which are empty. This is done for different mesh sizes. The patterns were found to be fractal over the entire size range.
Studying the paintings chronologically showed that the complexity of the fractal patterns, $D$, increased as Pollock refined his technique. One $D$ value is clearly an outlier—1.9 in 1950, a work that Pollock later destroyed. He may have thought this image was too dense or too complex and subsequently scaled back.
How to fold paper in half twelve times

[Historical Society of Pomona Valley, 2011, Weisstein, 2012]

The loss function for folding paper in half was derived by high school student Britney Gallivan in 2001. Britney then set a new world record by folding first gold foil and then paper in half 12 times in 2002, thus debunking the assertions of Math@Home and PBS Kids that paper cannot be folded in half more than eight times.
New triangular inequalities in the form of $s - a$, $s - b$, and $s - c$ [Tsai, 2010]

In 2010, 17-year-old Kang Ying Liu of Hawai‘i discovered nine new geometric formulas for describing triangle inequalities. For this, she took first place in the 53rd Hawai‘i State Science and Engineering Fair.
New triangle centers associated with a triad of simulated circumcircles

[Bigold, 2011, Kimberling, 2012]

In 2011, Kang Ying Liu discovered three new triangle centers. For this, she took first place in the 54th Hawai‘i State Science and Engineering Fair.

Two of these triangle centers have been named after her.

(The Encyclopedia of Triangle Centers has called triangle centers X(3598) and X(3599) the 1st and the 2nd Liu points, respectively.)
The slides can be found at

The paper can be found at

A one-page summary can be found at http://www.scribd.com/doc/36872171/Periodic-Billiard-Paths-in-Triangles
It’s your turn

Look for articles and books by mathematics popularizers such as Martin Gardner and Ian Stewart. Also look at Danica McKellar’s books and her website at

http://www.danicamckellar.com/

More examples of high-school students who have made original contributions in mathematics can be found at

http://math.stackexchange.com/questions/174009/

Other easily-understood open problems in mathematics can be found at

http://mathoverflow.net/questions/100265/
Pat Bigold.
St. Andrew’s Priory girls dominated state science/engineering fair.

José Nilo G. Binongo.
Who wrote the 15th book of Oz? An application of multivariate analysis to authorship attribution.

Historical Society of Pomona Valley.
Folding paper in half twelve times.

Clark Kimberling.
Encyclopedia of Triangle Centers.

Richard P. Taylor.
Order in Pollock’s chaos.

Fractal analysis of Pollock’s drip paintings.  

Michael Tsai.  
Hawaii teen whiz creates new math formulas.  
http://the.honoluluadvertiser.com/article/2010/Apr/21/ln/hawaii4210335.html

Carol Vogel.  
A Pollock is sold, possibly for a record price.  
http://www.nytimes.com/2006/11/02/arts/design/02drip.html?_r=1&adxnnl=1&ref=arts&adxnnlx=1163031599-revbGMuaIhdTP4qLonq8BA&oref=slogin

Eric W. Weisstein.  
Folding.  
http://mathworld.wolfram.com/Folding.html